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OPTIMUM PLOT SIZE DETERMINATION AND ITS APPLICATION TO CUCUMBER YIELD TRIALS

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SUMMARY

Methods of estimating Smith's b and, thereby, optimum plot size are compared from a theoretical viewpoint. For estimating b, generalized least squares is recommended over SMITH's (1938) original method and other methods because the points used to fit the required regression are correlated and have unequal variances.

Optimum plot size for once-over-harvest trials measuring yield (as number of fruits per plot) of pickling and fresh-market cucumbers (*Cucumis sativus* L.) was estimated to be 0.7 to 3.8 m^2 (0.5 to 2.5 m of row for rows 1.5 m apart) for conventional harvesting, and 1.0 to 5.6 m^2 (0.7 to 3.7 m of row) for simulated harvesting using paraquat to defoliate plots before evaluation. Estimates of optimum plot size were calculated from a number of uniformity trials differing in year (1982 or 1983), planting date (early or late), and field. The estimates were sufficiently stable to suggest that they have useful generality.

For multiple-harvest yield trials, optimum plot sizes for determining yield of pickling (expressed in $/ha or q/ha) or fresh-market cucumbers (i.e. USDA Fancy and No. 1 grade fruit combined or USDA Fancy, No. 1, and No. 2 grade fruit combined, in q/ha) were estimated from experimental data to be 6.4 to 10.3 m^2 (4.3 to 6.8 m of row).

INTRODUCTION

Many considerations enter into the design of a field experiment. Some are dictated by available resources, the requirements of equipment to be used, and other practical matters. Blocking is often introduced to account for patterned environmental variation (known or suspected), or to establish convenient management units. However, usually little attention is paid to optimizing plot size, that is, choosing plot size to minimize cost per unit of information. (In experimental design terminology, 'information' is the reciprocal of variance.) This is another aspect of dealing with environmental heterogeneity. It is an attempt to reconcile information on *un*patterned environmental variation with cost considerations.

Large numbers of families are usually evaluated in cucumber (*Cucumis sativus* L.)

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breeding trials, especially in the early stages of the inbreeding process. This increases the importance of experimental efficiency. Optimum-plot-size calculations often recommend different plot sizes than might otherwise have been used; when smaller plots are recommended, as is often the case, these calculations can lead to substantial savings in labor and other costs. However, a concern about the value of optimum-plot-size estimation is the degree to which optimum plot size depends on factors such as the particular field, year, season (planting date), or trait of interest.

Our objectives in this study were as follows: First, to provide a brief unified synopsis of the fragmented and often confusing literature on optimum-plot-size methodology. Second, to compare, from a theoretical viewpoint, available methods for estimating optimum plot size, and to recommend one. And third, to apply the recommended method to a variety of data sets for both pickling and fresh-market cucumbers, to investigate the stability of optimum-plot-size estimates over fields, years, seasons, crops, and harvesting methods, and, provided the estimates are reasonably stable, to suggest appropriate plot sizes for cucumber breeding trials.

MATERIALS AND METHODS

Data. All data were collected at the Horticultural Crops Research Station near Clinton, North Carolina. Uniformity trials were planted in different fields 22 July 1982 and 23 May 1983. Rows were seeded on raised, shaped beds 1.5 m apart center-to-center using 'Calypso' and 'Slicemaster' as the pickling and fresh-market cultivars, respectively. There were 6 rows of each cultivar, each row consisting of 25 plots, each 1.5 m long. Plots were thinned to 15 plants at the first-leaf stage, and harvested once-over 2 and 9 September 1982 and 12 and 14 July 1983 for pickling and fresh-market cucumber plots, respectively. All plots were harvested when approximately 10% of the fruits were oversized (diameter >51 mm for pickling cucumbers and diameter >60 mm for fresh-market cucumbers). MILLER & HUGHES (1969) found that that fraction of oversized fruit coincided with the optimum stage for once-over harvest of pickling cucumbers; we used the same criterion for harvesting fresh-market cucumbers to standardize crop management practices. The yield of each plot was measured as number of fruits.

In addition, for the once-over-harvest trials, two harvesting methods with different labor requirements (costs) were compared. The conventional method involves hand-pulling the plants and counting the fruits as they are separated from the vines. A faster method, proposed by WEHNER et al. (1984), is to defoliate the plants with paraquat (1,1'-dimethyl-4,4'-bipyridinium ion), making the fruits visible without additional labor.

Multiple-harvest yield trials were planted in different fields 19 April 1982 and 11 July 1983 for pickling and fresh-market cucumbers, respectively. Rows were seeded on raised, shaped beds 1.5 m apart center-to-center, with 7 plots per row, each plot 6 m long. Trials were run as randomized-complete-block designs with 3 blocks of 35 lines for pickling and 28 lines for fresh-market cucumbers. Plots were thinned to 60 plants per plot (50 plants per plot for 1983 fresh-market cucumbers), and harvested 6 times. For pickling cucumbers, yield was measured as $/ha and q/ha. Dollar value was calculated using $0.31, 0.14, and 0.09 per kg of grade 1, 2, and 3 pickling cucumbers, respectively. North Carolina grades are determined by diameter, with grade 1 being 0 to 26 mm, grade 2 being 27 to 38 mm, and grade 3 being 39 to 51 mm in diameter. Oversized fruits (diameter > 51 mm) are considered worthless because they are not used commercially. For fresh-market cucumbers, yield was measured as q/ha of USDA Fancy and No. 1 grade fruit combined, and as q/ha of Fancy, No. 1, and No. 2 grade fruit combined.

Estimation of Smith's b. Optimum-plot-size determination is founded on the 'law' proposed by SMITH (1938). A loose argument for Smith's 'law' is as follows: Given a uniformity trial where the basic units (the smallest plots defined) have variance V_1, the mean of a random sample of x units has variance

$$V_{\bar{x}} = V_1/x, \quad (1)$$

ignoring possible 'corrections' for the finite size of any uniformity trial and, therefore, the finite number of plots that could be formed. Suppose, instead, contiguous plots are combined. Because they are expected to be positively intercorrelated, means of groups of x contiguous units should have larger variance

$$V_{\bar{x}} = V_1/x^b \quad (2)$$

for some b, where $0 \leq b \leq 1$. When $b = 1$, equations (1) and (2) are identical. Taking logs of both sides, equation (2) becomes

$$\log(V_{\bar{x}}) = \log(V_1) - b\log(x), \quad (3)$$

the equation of a straight line describing the relationship between $\log(V_{\bar{x}})$ and $\log(x)$. Equation (3) cannot be derived in any mathematically-rigorous way. SMITH arrived at it by empirical observation, based on 44 uniformity trials on a wide array of crops. He observed that if small plots were combined to form larger plots of first one size (value of x) and then another and another, with $\hat{V}_{\bar{x}}$, the estimated (observed) variance among plot means, calculated in turn for each plot size (x), then the apparent relationship between $\log(V_{\bar{x}})$ and $\log(x)$ was quite linear, at least within the range of reasonable plot sizes. That observation, encapsulated in equation (3), has become known as Smith's 'law'. Despite its lacking a firm theoretical basis, Smith's 'law' has proven to be broadly applicable, and computer simulation by PEARCE (1976) has given the 'law' added credibility.

The parameter b in equations (2) and (3) is widely known as Smith's b. The slope of a regression of $\log(\hat{V}_{\bar{x}})$ on $\log(x)$ provides an estimate of $-b$; that is, the negative of the slope is an estimate of Smith's b.

Smith's b is often called the coefficient of soil heterogeneity, but it is more correctly viewed as an index of the degree of correlation between neighboring plots; $b \doteq 1$ indicates neighboring plots are uncorrelated, while b near 0 indicates they are highly correlated. The magnitude of b reflects, in part, environmental heterogeneity, but includes that stemming from all sources (e.g., soil, rain, irrigation, fertilizer and herbicide levels, and that inadvertently introduced by workers in the course of running the experiment). When there is considerable fine-grained environmental heterogeneity, one expects correlation between neighboring plots to be small and b to be large. As seen in equation (2), $V_{\bar{x}}$ will be reduced more effectively by increasing x when b is large;

that is, increasing plot size will be most beneficial when neighboring plots are nearly uncorrelated. When neighboring plots are highly intercorrelated (small b), the benefits of increasing plot size (x) will be slight. However, as BRIM & MASON (1959) noted, b can be large when an area is either extremely, but randomly, heterogeneous or extremely uniform, so b is not simply measuring environmental heterogeneity.

KOCH & RIGNEY (1951) showed that b can be estimated from certain types of experimental data (e.g., lattice, split-plot, and randomized-complete-block designs), providing an alternative to running a time-consuming and perhaps costly uniformity trial. HATHEWAY & WILLIAMS (1958) gave an equivalent, but more convenient, formulation of KOCH & RIGNEY's approach, and stressed its applicability to uniformity trials as well as experimental data. For uniformity trial data, variances of means of plots of several sizes can be estimated by overlaying a fully-nested design onto the uniformity trial and calculating the associated analysis of variance. Table 1 shows an example of the analysis of variance for a nested design overlayed on any one of the uniformity trials described in paragraph 1 of Materials and Methods. The 25th plot in each row has been omitted and each of the 6 rows taken as 3 8-unit plots, repeatedly halved to form subplots, subsubplots, and subsubsubplots of sizes 4, 2, and 1, respectively. Throughout this paper, plot size is taken to be length along a single row; in other contexts, plots of other shapes may be of interest and can be handled similarly.

From Table 1, variances of means of plots of size x = 8, 4, 2, and 1, respectively, are estimated as

$$\begin{aligned} \hat{V}_{\bar{8}} &= [SSA/17]/8 = MSA/8, \\ \hat{V}_{\bar{4}} &= [(SSA + SSB)/35]/4, \\ \hat{V}_{\bar{2}} &= [(SSA + SSB + SSC)/71]/2, \text{ and} \\ \hat{V}_{\bar{1}} &= [(SSA + SSB + SSC + SSD)/143]/1, \end{aligned} \quad (4)$$

providing 4 points for estimating b from the regression of $\log(\hat{V}_{\bar{x}})$ on $\log(x)$ without explicitly forming plots of each of several sizes (x) and recomputing $\hat{V}_{\bar{x}}$ for each size as SMITH proposed. The final divisor in $\hat{V}_{\bar{8}}$, for example, appears because MSA estimates 8 times the variance among means of plots of size 8, where $V_{\bar{8}}$ is 1 times that variance. For the 36 plots of size 4, for example, the calculation of $\hat{V}_{\bar{4}}$ uses the fact that the sum of squares among these plots can be obtained as (SSA + SSB), and the mean square with 35 df as (SSA + SSB)/35, and so forth.

No matter whether they are obtained by SMITH's (1938) or HATHEWAY & WILLIAMS' (1958) approach, the points used to estimate b present two problems, both recognized by SMITH. First, as plot size (x) increases, fewer plots can be formed, so the variance among means of larger plots is not as well estimated as for smaller plots. SMITH there-

Table 1. Analysis of variance for a nested design overlaid on a uniformity trial.

Source of variation	d.f.	Sum of squares	Mean square
Plots of size 8	18–1 = 17	SSA	MSA
Subplots of size 4 within plots	18(2–1) = 18	SSB	MSB
Subsubplots of size 2 within subplots	18(2) (2–1) = 36	SSC	MSC
Subsubsubplots of size 1 within subsubplots	18(2) (2) (2–1) = 72	SSD	MSD

fore recommended fitting the line used to estimate b by weighted least squares, weighting points by their degrees of freedom (i.e., approximately proportionally to the reciprocals of their variances). Second, because the *same* data are (at least effectively) recombined in different ways to estimate variances of plots of different sizes, the points used to fit the line are not independent. In fact, they may be highly correlated, and weighting by degrees of freedom alone fails to account for this.

The important contribution of HATHEWAY & WILLIAMS (1958) was showing how to obtain quite easily an estimate of the variance-covariance matrix of the $\log(\widehat{V}_{\bar{x}})$-values for the several values of x, *provided* those $\widehat{V}_{\bar{x}}$'s are calculated as in Table 1 and equations (4). Their approach exploits the independence of sums of squares like those shown in Table 1, the ease with which their variances can be calculated under the usual normality assumption, and the way the $\widehat{V}_{\bar{x}}$'s are obtained in equations (4) as simple linear combinations of the sums of squares.

Using HATHEWAY & WILLIAMS' (1958) estimate of the variance-covariance matrix of the $\log(\widehat{V}_{\bar{x}})$'s, generalized least squares (GLS) can be used to calculate an estimate of b that will be approximately the minimum variance unbiased estimate (often called the best linear unbiased estimate or b.l.u.e.). There are both theoretical and practical reasons for preferring GLS in estimating Smith's b and, ultimately, optimum plot size. When the variance-covariance matrix is known, AITKEN's (1935) generalization of the Gauss-Markov theorem on the theory of least squares assures us that GLS is optimal. Even when the variance-covariance matrix must be estimated, provided it can be estimated reasonably well, GLS still should be preferable to weighted (WLS) or ordinary (OLS) least squares. Admittedly, when the same data are used in GLS to estimate b as to estimate the variance-covariance matrix (the usual case), the GLS estimator is not unbiased, but the bias should be very slight. More importantly, because the GLS estimator of b takes account both of the (very) unequal variances and of the (large) covariances of the points used in estimating b, the GLS estimator should have smaller mean squared error (variance plus squared bias) than the WLS or OLS estimator, and be closer to the true (unknown) value of b. WLS and OLS estimators, calculated under implicit (false) assumptions that the points are uncorrelated and, for OLS, have equal variance, will be inefficient. Relative to the standard errors of GLS estimates, those for WLS estimates will be inflated, and for OLS estimates even more inflated. (See the Appendix for more mathematical discussion and comparison of these estimators.)

As a practical matter, more variable estimators are also more likely to give estimates of b that are outside the allowable range 0 to 1, as one would expect and as has been borne out in our experience and that of others (e.g., BRIM & MASON, 1959). Values outside the range 0 to 1 cannot be used to estimate optimum plot size by equation (7) below.

For both theoretical and practical reasons, therefore, we recommend the GLS estimator. From our uniformity trial data we estimate b by GLS using plot sizes $x = 1, 2, 4$ and 8 with plots formed and variances calculated as illustrated in Table 1 and equations (4). Although the estimates of b do not depend on whether common (base 10) or natural logs are used in the calculations, the reader is warned that the estimated variance-covariance matrices used for GLS in the two cases do differ by a constant multiple.

For estimating b with data from a designed experiment, the procedure outlined above for uniformity trials must be modified slightly to take account of the fact that some degrees of freedom are used to estimate treatment effects (HATHEWAY & WILLIAMS, 1958; KOCH & RIGNEY, 1951). When the available data are from a randomized complete block design, variances can be estimated for only 2 plot sizes, the basic plot size in the design ($x = 1$) and the block size ($x = 28$ or 35 for our data). Block-treatment interaction must be assumed to be negligible. With only 2 points, the method of fitting the line is irrelevant (fitted by any method, the line must pass through both points), and no standard error can be calculated for the estimate of b. More complex designs (e.g., split plots and lattices) may provide 3 or more points for fitting the line by GLS and permit calculating standard errors of the estimates (see HATHWAY & WILLIAMS, 1958, and KOCH & RIGNEY, 1951, for examples).

BINNS (1982) and SMITH (1938) discuss modifying the calculations for estimating $\hat{V}_{\bar{x}}$ and b when experimental plots are to be arranged in blocks (with added efficiency from intrablock correlation), and to account for the finite size of the area providing data for estimation. However, the importance of these 'correction factors' in estimating b is diminished under GLS since, as HATHEWAY & WILLIAMS (1958) note, the terms most affected are those given smallest weight by the GLS procedure. Furthermore, the effect of these adjustments becomes less as the number of plots per block increases, and that number is likely to be large in cultivar yield trials. For both reasons, we omit these complicating correction factors from all our calculations.

Optimum plot size determination. Having estimated b, to estimate optimum plot size one must introduce cost considerations. Although it is always true that larger plots provide more information per plot than smaller plots, the apparent advantage of larger plots fades when costs are taken into account. If costs of a plot are partitioned into

K_1 = cost per plot for costs that do not depend on plot size, and
K_2 = cost per unit area for costs that increase with plot size
= additional cost for each unit increase in plot size, then

$$\text{total cost of a plot of size } x = K_1 + K_2x. \tag{5}$$

The size of the unit depends on the data to be used. It was 1.5 m and 6 m of row length for our uniformity trial and experimental data, respectively. K_1 and K_2 can be obtained by summing costs in the two categories, or as the intercept and slope, respectively, of the line relating total cost of a plot (y-axis) to size of the plot, expressed in number of units (x-axis).

From equations (2) and (5),

$$\text{information} = 1/\text{variance} = 1/(V_1/x^b) = x^b/V_1,$$

so cost per unit of information is

$$(K_1 + K_2x)/(x^b/V_1) = V_1(K_1 + K_2x)/x^b. \tag{6}$$

Setting the derivative of (6) with respect to x equal to zero and solving, one finds that x_{opt}, the value of x that minimizes cost per unit of information, is

$$x_{opt} = bK_1/[(1-b)K_2], \tag{7}$$

and is in the same plot-size units used in determining K_1 and K_2, and in estimating b. That is to say, x_{opt} is some multiple of the basic plot size. For example, $x_{opt} = 2$ means that the optimum plot size is twice the size of the smallest plot used in estimating b. No matter how b is estimated, equation (7) is used to calculate x_{opt}. Equation (7) shows that the optimum plot size is an increasing function both of b and of the ratio of costs K_1/K_2. This agrees with intuition. When b is very small (neighboring plots highly correlated), little is gained by combining small plots to form large ones; the gain in information is unlikely to be commensurate with the increase in costs. In the extreme case wherein $b = 0$ (variances and covariances of neighboring plots all equal), the $\hat{V}_{\bar{x}}$ of equations (4) are equal in expectation for all x, that is, there is zero increase in information with increased plot size. In the other extreme case wherein $b = 1$ (neighboring plots uncorrelated), doubling the plot size should halve the variance among plot means and thereby double the information per plot as shown in equations (1) and (2), but at (presumably) less than double the cost, so larger plots are advantageous. With respect to K_1/K_2, when $K_1 << K_2$ (small ratio), costs are largely proportional to plot size, favoring smaller plots. When $K_1 >> K_2$ (large ratio), there is relatively little added cost in increasing plot size, favoring larger plots.

One can see from equation (7) that it is especially important when b is large to have as good an estimate of b as possible. When b is large, the estimate of optimum plot size is particularly sensitive to minor differences in the estimate of b, which are amplified in $b/(1-b)$.

RESULTS AND DISCUSSION

Once-over-harvest trials. Table 2 gives estimated costs for determining number of fruits per plot in a once-over harvest for both conventional hand-pulled plots and paraquat-defoliated plots. Table 3 displays the GLS estimates of b obtained from the 4 uniformity trials, the standard errors (S.E.'s) of these estimates, and the estimates of x_{opt} and optimum plot size obtained using the GLS estimates of b and the costs from Table 2.

Table 2. Labor costs (worker-hours) for a once-over-harvest trial measuring yield in pickling or fresh-market cucumbers using 2 harvesting methods[1].

	Conventional hand-pulled plots		Paraquat-defoliated plots	
Operation	K_1	K_2	K_1	K_1
Field plan/Data sheets	0.008	0	0.008	0
Seed packeting/Stakes	0.007	0	0.007	0
Field layout/Planting	0.012	0.036	0.012	0.036
Thinning/Stand counting	0.005	0.024	0.005	0.024
Harvesting/Data collection	0.011	0.060	0.011	0.021
Data analysis	0.007	0	0.007	0
Subtotal	0.050	0.120	0.050	0.081
Total	0.170		0.131	

[1] Basic plot size is $2.25\ m^2$ (1.5 m of row for rows 1.5 m apart).

Table 3. Estimates of b, x_{opt}, and optimum plot size for yield (i.e., number of fruits per plot) in once-over harvest of pickling or fresh-market cucumbers, based on uniformity trial data[1].

Cucumber type	Year	GLS estimate[2] of b	S.E. of GLS estimate	Conventional harvesting		Defoliation harvesting	
				x_{opt}[3]	est. of opt. plot size (m^2)	x_{opt}[3]	est. of opt. plot size (m^2)
Pickle	1982	0.523	0.111	0.46	1.0	0.68	1.5
	1983	0.801	0.118	1.68	3.8	2.48	5.6
Fresh-market	1982	0.615	0.104	0.67	1.5	0.99	2.2
	1983	0.419	0.105	0.30	0.7	0.45	1.0

[1] Basic plot size is 2.25 m^2 (1.5 m of row for rows 1.5 m apart).
[2] Obtained by estimating variances of means of plots of different sizes by the method of HATHEWAY & WILLIAMS (1958), with b estimated by generalized least squares (GLS).
[3] Multiple of basic plot size.

Estimates of optimum plot size for conventional once-over harvesting of pickling and fresh-market cucumbers were found to range from 0.7 to 3.8 m^2 (0.5 to 2.5 m of row). Estimates of optimum plot size under the simulated once-over harvesting using paraquat to defoliate the plots before evaluation were predictably higher, consistent with reduced K_2 (costs dependent on plot size) and thus increased cost ratio K_1/K_2. For herbicide-defoliated plots, estimates of optimum plot size range from 1.0 to 5.6 m^2 (0.7 to 3.7 m of row).

The optimum-plot-size estimates reported in Table 3 vary surprisingly little, considering that the trials differed in crop, year, field, and season. They are sufficiently stable to suggest that estimates of optimum plot size for cucumbers have useful generality. This is true, in part, because one has some latitude in the choice of plot size. SMITH (1938) showed graphically that using plot sizes within the range 0.25 (x_{opt}) to 4(x_{opt}) will be at least 75% cost-efficient for values of b likely to occur in practice, where cost-efficiency is the relative cost per unit of information obtained with the optimum plot size and with another plot size. Beyond this range, cost-efficiency can drop quite quickly. So, because estimates of b often have large standard errors, it is still important to use the best estimator available. When b is small, one should especially avoid using excessively large plots. When b is large, plots that are much smaller than the optimum size are most disadvantageous (cost-inefficient).

Our estimates of optimum plot size are consistent with the estimate 3.6 m^2 reported by SMITH & LOWER (1978) for evaluating yields of pickling cucumbers by once-over harvest. Their data were from a single field planted in spring 1975. They used KOCH & RIGNEY's (1951) method (which uses ordinary least squares in estimating b) with plot sizes x = 1, 3, and 27 single-row sections 2.4 m long (1.5 m between rows), but noted that the point calculated for x = 27 fell below the line through the other two points. We initially used plot sizes x = 1, 2, 4, 8 and 24; when the point for plot size x = 24 (whole row) was included, the standard errors of even the GLS estimates of b often increased by 25 to 35%, despite the fact that the GLS procedure assigned this point smallest weight in the calculations. It seems advisable to restrict attention to reasonable plot sizes, not just because they are realistic, but because so few very large plots can be formed that (i) the variance among their means is poorly estimated, and (ii) omitting 'finite population correction factors' from the calculations (BINNS, 1982; SMITH, 1938) is probably not justifiable.

Multiple-harvest trials. Tables 4 and 5 give labor costs, estimates of b, and estimates of x_{opt} and optimum plot size for multiple-harvest trials to determine $/ha and q/ha for pickling cucumbers, and q/ha of USDA Fancy and No. 1 grade fruit combined and q/ha of USDA Fancy, No. 1, and No. 2 grade fruit combined for fresh market cucumbers. As discussed earlier, estimates of b are based on only 2 plot sizes ($x = 1$ and $x = 28$ or 35, here) for data from randomized-complete-block design yield trials. No standard errors can be calculated with only 2 plot sizes, and, based on our experience and that of SMITH & LOWER (1978) with very large plot sizes (see preceding paragraph), these estimates of b should be viewed as rough estimates. In fact, the estimates of b from the 1982 fresh-market cucumber data exceed 1, and x_{opt} is not calculated. The calculations reported in Table 5 recommend plot sizes of 6.4 to 10.3 m^2 (4.3 to 6.8 m of row for rows 1.5 m apart) for multiple-harvest yield trials such as these.

In general, different dependent variables may have different optimum plot sizes, although that is not very evident in Table 5 because the dependent variables we considered are highly correlated. When there are several dependent variables of interest, one may have to estimate optimum plot size for each of the variables, and then use a compromise plot size.

Final comments and conclusions. The cucumber-breeding program at North Carolina State University now uses 2 replications of plots that are 1.5 m of row (rows 1.5 m apart) for testing in early generations. This is determined in part by limited seed supplies, using 60 seeds of the approximately 100 seeds obtained from each family. For multiple-harvest trials, plots that are 6 m of row (rows 1.5 m apart) are now used in 2 seasons with 3 replications in each. These plot sizes fall within the ranges of optimum-plot-size estimates obtained here, and provide sufficient numbers of fruits for

Table 4. Labor costs (worker-hours) for a multiple-harvest trial measuring yield in pickling and fresh-market cucumbers[1].

Operation	Pickling cucumbers		Fresh-market cucumbers	
	K_1	K_2	K_1	K_2
Field plan/Data sheets	0.037	0	0.037	0
Seed packeting/Stakes	0.029	0	0.029	0
Field layout/Planting	0.099	0.053	0.099	0.053
Thinning/Stand counting	0.004	0.015	0.004	0.015
Harvesting	0.113	0.230	0.102	0.207
Grading/Weighing/Analysis	0.200	0.086	0.206	0.094
Subtotal	0.482	0.384	0.477	0.369
Total		0.866		0.846

[1] Basic plot size is 9 m^2 (6 m of row for rows 1.5 m apart).

Table 5. Estimates of b, x_{opt}, and optimum plot size for multiple harvest of pickling and fresh-market cucumbers, based on experimental data[1].

Cucumber type	Year	Measure of yield[2]	Estimate of b	x_{opt}[3]	Estimate of optimum plot size (m^2)
Pickle	1982	\$/ha	0.475	1.14	10.3
		q/ha	0.362	0.71	6.4
	1983	\$/ha	0.428	0.94	8.5
		q/ha	0.393	0.81	7.3
Fresh-market	1982	q/ha (F+1)	1.606	–	–
		q/ha (F+1+2)	1.135	–	–
	1983	q/ha (F+1)	0.458	1.09	9.8
		q/ha (F+1+2)	0.432	0.98	8.8

[1] Basic plot size is 9 m^2 (6 m of row for rows 1.5 m apart).
[2] For pickling cucumbers, \$/ha was calculated using values \$0.31, 0.14, and 0.09 per kg of grade 1, 2, and 3 fruit, respectively. Fresh-market cucumbers were graded as Fancy (F), No. 1 (1), and No. 2 (2) before weighing.
[3] Multiple of basic plot size.

determination of all of the characters measured in the trials. Practical considerations (e.g., limited seed supplies, requirements of equipment) always influence experimental planning. However, when feasible, it seems only sensible to estimate economical or optimum plot size, and to consider that factor too in the planning process.

We recommend the generalized least squares (GLS) approach of HATHEWAY & WILLIAMS (1958) for use in estimating Smith's b and, thereby, optimum plot size. Its merits warrant the added complexity. In some applications, the estimates of optimum plot size will not be very good, no matter what method of estimation is used. In such cases, one may be tempted to argue that, if you cannot obtain a good estimate anyway, you might as well use an easier method. On the contrary, the importance of using the best method available is greatest then. Because the points fitted in estimating b have unequal variances and are correlated, the theoretical appropriateness and practical advantages of GLS justify the extra computation it requires.

APPENDIX

Familiarity with matrix algebra is increasingly widespread (see, e.g., SEARLE (1966)), and it is helpful in making a concise comparison of the OLS, WLS, and GLS estimators of Smith's b, and in clarifying some of the required computations. The following discussion is for the case where b is to be estimated from uniformity trial data with the variances of means of plots of different sizes estimated as suggested by KOCH & RIGNEY (1951) and HATHEWAY & WILLIAMS (1958), and as illustrated in the example of Table 1 and equations (4). The same example will be continued. If the variances are to be estimated from experimental rather than uniformity trial data, minor modifications are necessary to account for the fact that some degrees of freedom are used to estimate treatment effects; these modifications are discussed by KOCH & RIGNEY (1951) and by HATHEWAY & WILLIAMS (1958).

If S plot sizes are being considered (S = 4 in our example), we can write

$$\mathbf{Y}_{Sx1} = \{\log(\hat{V}_{\bar{x}})\} \qquad \text{for } x = 8, 4, 2, 1$$

$$= \begin{bmatrix} \log(\hat{V}_{\bar{8}}) \\ \log(\hat{V}_{\bar{4}}) \\ \log(\hat{V}_{\bar{2}}) \\ \log(\hat{V}_{1}) \end{bmatrix} \qquad \text{for the example,}$$

$$\mathbf{X}_{Sx2} = [\,\mathbf{1} \quad \{\log(x)\}] \qquad \text{for } x = 8, 4, 2, 1$$

$$= \begin{bmatrix} 1 & \log(8) \\ 1 & \log(4) \\ 1 & \log(2) \\ 1 & \log(1) \end{bmatrix} \qquad \text{for the example,}$$

1 being an Sx1 vector of 1's. The vector of parameters is

$$\mathbf{b}_{2x1} = \begin{bmatrix} a \\ b \end{bmatrix},$$

where a and b are the intercept and slope, respectively, of the line described by equation (3). The negative of the slope b, the second element of **b**, is Smith's b which we wish to estimate.

The OLS estimator of the slope b, recommended by KOCH & RIGNEY (1951), is the second element of

$$\hat{\mathbf{b}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}.$$

This estimator takes account of neither the unequal variances nor the covariances among the elements of **Y**.

For WLS, weighting each observation by its degrees of freedom, we can define the SxS diagonal matrix of degrees of freedom by

$$\mathbf{W}_{SxS} = \mathbf{D}\{df_i\} \quad \text{for } i = 1,\dots,S$$

$$= \begin{bmatrix} 17 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 \\ 0 & 0 & 36 & 0 \\ 0 & 0 & 0 & 72 \end{bmatrix} \qquad \text{for the example.}$$

For observations (elements of **Y**) which themselves are logarithms of variances, weighting by degrees of freedom is, to a first approximation, weighting each observation proportionally to the reciprocal of its variance. The WLS estimator of the slope b is then the second element of

$$\hat{\mathbf{b}}_{WLS} = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{Y}.$$

This still fails to take account of the non-independence (covariances) of the elements of **Y**.

HATHEWAY & WILLIAMS (1958) provide a $\hat{\mathbf{V}}^{-1}$, an approximation (estimate) of $\mathbf{V}^{-1}$, the inverse of the variance-covariance matrix of the observations. For our example, provided natural logarithms are used throughout, this is the 4×4 matrix HATHEWAY & WILLIAMS show at the bottom of their p. 212, modified as directed in the first sentence of p. 213. As HATHEWAY & WILLIAMS note, an easy and important check that $\hat{\mathbf{V}}^{-1}$ has been correctly calculated is to verify that the elements of $\hat{\mathbf{V}}^{-1}$ sum to half the corrected total degrees of freedom [(144–1)/2 = 71.5, for our example]. The approximate GLS estimator of the slope b is the second element of

$$\hat{\mathbf{b}}_{GLS} = (\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{Y}.$$

The 2x2 variance-covariance matrix of the elements of $\hat{\mathbf{b}}_{GLS}$ is estimated by

$$\text{var}(\hat{\mathbf{b}}_{GLS}) = (\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X})^{-1},$$

with the standard error of the GLS estimate of Smith's b being the square root of the lower-right-hand element of $(\mathbf{X}'\hat{\mathbf{V}}^{-1}\mathbf{X})^{-1}$. Only the GLS estimator of Smith's b takes account of both the unequal variances of the points used to estimate b and, through the non-zero off-diagonal elements of $\hat{\mathbf{V}}^{-1}$, their lack of independence.

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